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SOUND RADIATION BY A PLATE REINFORCED BY A SET OF PROJECTING STIFFENER RIBS SUBJECTED TO A PERIODIC SYSTEM OF FORCES*

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There is considered sound radiation by a plate reinforced by a periodic set of projecting stiffener ribs of small wave dimensions. The source of the field is a set of lumped harmonic forces applied to the plate periodically in each transit, where the forces are taken identical in amplitude but with a constant phase shift between adjacent sources. The problem reduces to a quasiregular infinite system of linear algebraic equations, in terms of whose solution the energy fluxes leaving the plate into the fluid are expressed.

Sound radiation by periodically reinforced plates has been studied in many papers (see /1/, etc.). A common simplification was not taking account of sound reflection from the rib surfaces, in other words, considering them to influence only the conditions at which oscillations of the carrying plate occur.

This paper continues the investigation /2,3/ on the influence of the stiffener rib reflecting surface on the diffraction field. However, the method developed there permits computation of the radiation of the construction under consideration only for a sufficiently small ratio between the rib height and the spacing between them. A more realistic case, when this ratio is on the order of one, is investigated below. The factorization method /4/ is used in a form analogous to the constructions in /5/, where short wave diffraction is considered by a plate reinforced by one rib. Amplitudes of the waves being propagated over the ribs and their energy fluxes are found. The dependence of the fluxes on the frequency and point of application of the force is studied. The limits of applicability of an approximate examination of diffraction processes without taking account of sound reflection from the rib surfaces are discussed.

1. Let a plate $\{-\infty < x < \infty,\ y = 0\}$ be reinforced by projecting stiffener ribs $\{x = nd,\ 0 < y < h\}(-\infty < n < \infty)$. The structure is excited by a set of point forces

$$f(x) = F\sum_{n=-\infty}^{\infty}\delta(x - x_0 - nd)\exp(in\alpha) \tag{1.1}$$

applied to the plate $(0 < x_0 < d)$. The phase shift α can be considered to vary between the limits $|\alpha| \leqslant \pi$. The problem is planar, the time dependence of the processes $\exp(-i\omega t)$ (ω is the frequency of oscillation) is omitted. The pressure $p(x, y)$ $(y > 0)$ in the fluid satisfies the Helmholtz equation for the boundary conditions on the plate and the rib surfaces /2,3/

$$Lp(x,0) = \left[\left(\frac{\partial^4}{\partial x^4} - k_0^4\right)\frac{\partial}{\partial y} + \nu\right]p(x,0) = \nu f(x) + \sum_{-\infty}^{\infty} B_n\delta(x - nd) + C_n\delta'(x - nd) \quad (-\infty < x < \infty) \tag{1.2}$$

$$p_x(nd,\ y) + yp_{yx}(nd,0) = 0 \quad (0 < y < h, -\infty < n < \infty) \tag{1.3}$$

The boundary contact constants B_n, C_n are determined from the conditions of rib juncture with the plate

$$Z_1p_y(nd,\ 0) = [p_{yxxx}(nd,\ 0)]$$
$$-Z_2p_{yx}(nd,0) = [p_{yxx}(nd,0)] + \nu\int_0^h s\,[p(nd,s)]\,ds \tag{1.4}$$

These conditions, as well as the meaning of the parameters k_0, ν, Z_1, Z_2 are discussed in /2,3/. The symbol $[\varphi(nd)]$ denotes a jump in the function $\varphi(x)$ as it passes through the point $x = nd$. Finally, the pressure field $p(x, y)$ is constructed in conformity with the principle

*Prikl.Matem.Mekhan.,Vol.46,No.6,pp.1001-1010,1982 of limit absorption /6,7/ and should satisfy the Meixner condition "on the rib" /8/.

2. In conformity with the almost periodicity of the system of forces (1.1) applied to the plate, we shall seek the pressure field in the form of an almost-periodic function, i.e., we assume that

$$p(x+md,\ y) = p(x,\ y)\exp(im\alpha) \tag{2.1}$$

To give a foundation to the scheme for application of the limit absorption principle, it should be established that the boundary value problem for the Helmholtz equation has a unique solution under the conditions (1.2) - (1.4) and (2.1) in the presence of absorption in the medium ($\operatorname{Im} k > 0$). It is here considered that the field $p(x, y)$ satisfies the Meixner condition and decreases sufficiently rapidly at infinity, to assure convergence of the integrals occurring below. We note that uniqueness of the solution is proved rigorously in /9/ for problems of electromagnetic wave diffraction by arrays.

Let us use the second Green's formula for the pair of functions $p(x,\ y)$ and $\bar{p}(x, y)$ (the bar is the symbol of the complex conjugate, and ρ_0 is the fluid density) in the domain Ω (S is the boundary of the domain Ω)

$$\frac{1}{2\rho_0\omega}\int_\Omega (\bar{p}\Delta p - p\Delta\bar{p})\,d\Omega = \frac{1}{2\rho_0\omega}\int_S \left(\frac{\partial p}{\partial n}\bar{p} - \frac{\partial\bar{p}}{\partial n}p\right)dS \tag{2.2}$$

$$S = \{0 < x < d,\ y = 0\} \cup \{0 < x < d,\ y = H\}$$
$$\cup\ \{x = +0,\ 0 < y < H\} \cup \{x = d - 0,\ 0 < y < H\}$$

Taking account of the Helmholtz equation, there follows from the identity (2.2)

$$-\frac{\operatorname{Im} k^2}{2\rho_0\omega}\int_\Omega |p|^2\,d\Omega = \frac{1}{2\rho_0\omega}\operatorname{Im}\int_S \frac{\partial p}{\partial n}\bar{p}\,dS \tag{2.3}$$

The left side here yields the energy absorbed in the domain Ω, and the right side the energy flux through the boundary S /7/. Integration by parts with the whole set of boundary and boundary-contact conditions and the Meixner condition, as well as the substantiality of parameters k_0, ν, Z_1, Z_2 are real, taken into account is executed in transformation of the right side. We just present the result

$$-\frac{\operatorname{Im} k^2}{2\rho_0\omega}\int_\Omega |p|^2\,d\Omega = -\operatorname{Im}(\bar{F}p_y(x_0, 0)) + \frac{1}{2\rho_0\omega}\operatorname{Im}\int_0^d p_y(x, H)\,\bar{p}(x, H)\,dx \tag{2.4}$$

We assume that as $H\to\infty$ the field $p(x, H)\to 0$ together with $p_y(x, H)$, where L_2, the norm in the left side of (2.4), remains finite. We find

$$\frac{\operatorname{Im} k^2}{2\rho_0\omega}\lim_{H\to\infty}\int_\Omega |p|^2\,d\Omega = \frac{1}{2\rho_0\omega}\operatorname{Im}(\bar{F}p_y(x_0, 0)) \tag{2.5}$$

Under the coniditions of the homogeneous problem $F = 0$, from which $p(x,\ y)\equiv 0$, which indeed proves the uniqueness of the solution.

In the absence of absorption, we arrive at the identity

$$\frac{1}{2\rho_0\omega}\operatorname{Im}\int_0^d p_y(x, H)\,\bar{p}(x, H)\,dx = \frac{1}{2\rho_0\omega}\operatorname{Im}(\bar{F}p_y(x_0, 0)) \tag{2.6}$$

Here the right side describes the energy inserted into the structure, while the left side is the energy flux through the contour $\{0 < x < d,\ y = H\}$. Because of the almost-periodicity condition (2.1) in the space above the ribs $(y > h)$, the pressure field is representable in the form of the Fourier series

$$p(x,\ y) = \sum_{-\infty}^{\infty} A_n \exp(i\lambda_n x - \gamma_n y) \tag{2.7}$$

$$\lambda_n = \frac{2\pi n + \alpha}{d},\quad \gamma_n = \sqrt{\lambda_n^2 - k^2},\quad \operatorname{Re}\gamma_n \geqslant 0$$

Taking also account of the adhesion condition for the plate displacement $\zeta(x) = p_y(x, 0)/(\rho_0\omega^2)$ /1/, we reduce the identity (2.6) to the form

$$\frac{d}{2\rho_0\omega}\sum |A_n|^2\sqrt{k^2 - \lambda_n^2} = \frac{\omega}{2}\operatorname{Im}(\bar{F}\zeta(x_0)) \tag{2.8}$$

Summation here is just over the waves being propagated, for which $|\lambda_n| < k$. The identity (2.8) is the energy conservation law and can be used to check the correctness of computations of the structure radiation. It agrees with the identity found in /9/ for the case of plane electromagnetic waves diffraction by arrays. The right side of (2.8) is hence a quantity that can be measured directly on a plate.

Under the conditions of the homogeneous problem, the identity (2.8) shows that the amplitudes of all the waves being propagated above the ribs are zero (with the possible exception of the "slipping wave" for which $|2\pi n + \alpha| = kd$). However, it is impossible to assert this with respect to the waves not being propagated, and therefore, a solution of the type of waves traveling along the structure and decreasing exponentially with distance away generally appears at certain frequencies in the homogeneous problem. In the absence of ribs, such a wave is a surface wave /1/ at those frequencies at which its wave number $\varkappa$ satisfies the condition $|2\pi m + \alpha| = \varkappa d$ with some integer m. We still note that non-uniqueness of the solution is reduced by inserting absorption in the structure material even for $\operatorname{Im} k = 0$.

3. In the zero-th approximation, when sound diffraction by the stiffener rib surface is not taken into account, the pressure field is found by elementary means in the form of a series of the type (2.7) (the lack of limits to the summation below means that it is executed over all integer n)

$$p_0(x, y) = -\frac{\nu F}{d} \sum \frac{1}{L_n} \exp(i\lambda_n x - \gamma_n y)(\exp(-i\lambda_n x_0) + f_1 + i\lambda_n f_2), \quad L_n = (\lambda_n^4 - k_0^4)\gamma_n - \nu \tag{3.1}$$

$$f_1 = \frac{1}{\Delta(\alpha)}\left\{U_1(0)U_1(-x_0) - \left(U_2(0) + \frac{d}{Z_2}\right)U_0(-x_0)\right\}$$

$$f_2 = \frac{1}{\Delta(\alpha)}\left\{U_1(0)U_0(-x_0) - \left(U_0(0) - \frac{d}{Z_1}\right)U_1(-x_0)\right\}$$

$$\Delta(\alpha) = \left(U_0(0) - \frac{d}{Z_1}\right)\left(U_2(0) + \frac{d}{Z_2}\right) - U_1^2(0)$$

$$U_s(x) = \sum \frac{\gamma_n (i\lambda_n)^s}{L_n} \exp(i\lambda_n x) \quad (s = 0, 1, 2)$$

The integral term in (1.4) is discarded and condition (1.3) is not taken into account in constructing the field p_0. Such a simplification is characteristic for all preceding work, and one of the problems of this investigation is to discuss its competence.

The field $q = (p - p_0)/(\nu F)$, associated with diffraction by the stiffener rib surface, satisfies the condition on the plate

$$Lq(x, 0) = \sum B_n \delta(x - nd) + C_n \delta'(x - nd) \tag{3.2}$$

the condition on the rib surface $(-\infty < m < \infty)$

$$q_x(md, y) + y q_{yx}(md, 0) = g(y)\exp(im\alpha) \quad (0 < y < h) \tag{3.3}$$

$$g(y) = \frac{1}{d}\sum \frac{i\lambda_n}{L_n}\exp(-\gamma_n y)(\exp(-i\lambda_n x_0) + f_1 + i\lambda_n f_2) - \frac{y}{Z_2} f_2$$

and the homogeneous boundary-contact condition (1.4). For the future it is convenient to write the function $g(y)$ in the form of an integral by using an analog of the Watson transformation

$$g(y) = \frac{1}{2\pi}\int_\Lambda \frac{i\lambda e^{i\lambda y}}{l(\lambda)S(\lambda)}(G(\lambda, x_0) + f_1 c(\lambda) + f_2 \gamma \operatorname{sh}\gamma d)\, d\lambda + \frac{y}{Z_2} f_2 \tag{3.4}$$

$$\gamma = \sqrt{\lambda^2 - k^2}, \quad S(\lambda) = \operatorname{ch}\gamma d - \cos\alpha, \quad G(\lambda, x_0) = \operatorname{ch}\gamma(x_0 - d) - e^{-i\alpha}\operatorname{ch}\gamma x_0 \quad c(\lambda) = e^{i\alpha} - \operatorname{ch}\gamma d, \quad l(\lambda) = i\lambda(\lambda^4 - k_0^4) + \nu$$

Here the contour Λ is centrally symmetric relative to $\lambda = 0$, passes along the real axis for large λ while remaining below the zeroes of the polynomial $l(\lambda)$ and the zeroes $\lambda = -i\gamma_n$ of the function $S(\lambda)$. Contours of this kind occur systematically in investigations on acoustic diffraction by structures from intersecting plates /5,10/.

The almost-periodic dependence of the field q on the coordinate x permits /4/ reduction of the problem in a half-space to a problem in a half-strip $\{0 < x < d, y > 0\}$. In conformity with the representation (3.4), we will seek the field in the form (below the contour of integration is Λ)

$$q(x, y) = \int \frac{u(\lambda)\operatorname{ch}\gamma x + v(\lambda)\operatorname{sh}\gamma x}{l(\lambda)S(\lambda)} e^{i\lambda y}\, d\lambda$$

with the unknown functions u and v. The boundary condition on the plate results in the equation

$$\int \frac{u(\lambda)\,\mathrm{ch}\,\gamma x + v(\lambda)\,\mathrm{sh}\,\gamma x}{S(\lambda)}\,d\lambda = 0 \quad (0 < x < d)$$

to satisfy which it is sufficient to consider the functions $u(\lambda)$ and $v(\lambda)$ odd.

Furthermore, from the continuity condition of the derivative $q_x(0, y)$ for all $y > 0$ we have

$$[q_x(0, y)] = q_x(+0, y) - e^{-i\alpha} q_x(d-0, y) = \int \gamma \frac{Q(\lambda)}{l(\lambda)\,S(\lambda)} e^{i\lambda y}\,d\lambda = 0$$

$$Q(\lambda) = v(\lambda) - e^{-i\alpha}(u(\lambda)\,\mathrm{sh}\,\gamma d + v(\lambda)\,\mathrm{ch}\,\gamma d)$$

To satisfy the last equation it is sufficient to set

$$\gamma Q(\lambda) = l(\lambda)\,S(\lambda)\,\Phi^+(\lambda); \quad \Phi^+(\lambda) \in W^+(1+\varepsilon_1), \quad \varepsilon_1 > 0 \qquad (3.5)$$

Here and henceforth $W^\pm(a)$ denote the classes of function analytic above (below) Λ with the asymptotic $O(\lambda^{-a})$ as $\lambda \to \infty$. The form of the asymptotic here follows from the Meixner condition /4/.

From the oddness of u and v we find

$$l(\lambda)\Phi^+(\lambda) = -l(-\lambda)\,\Phi^+(-\lambda), \quad \lambda \in \Lambda$$

Taking account of the Liouville theorem, analytic continuation in the last relationship yields

$$l(\lambda)\Phi^+(\lambda) = p_3(\lambda) \qquad (3.6)$$

with an arbitrary odd polynomial of third degree $p_3(\lambda)$.

Taking account of (3.6), we write the general solution of the functional equation (3.5) in the form

$$u(\lambda) = \frac{p_3(\lambda)}{\gamma}\,\mathrm{sh}\,\gamma d + p(\lambda)\,c(\lambda), \quad v(\lambda) = \frac{p_3(\lambda)}{\gamma}\,c(\lambda) + p(\lambda)\,\mathrm{sh}\,\gamma d$$

with the new unknown odd function $p(\lambda)$. Finally we arrive at the following representation

$$q(x, y) = \int \frac{e^{i\alpha}\,\mathrm{ch}\,\gamma x - \mathrm{ch}\,\gamma(x-d)}{l(\lambda)\,S(\lambda)}\,p(\lambda)\,e^{i\lambda y}\,d\lambda + \int \frac{e^{i\alpha}\,\mathrm{sh}\,\gamma x - \mathrm{sh}\,\gamma(x-d)}{\gamma l(\lambda)\,S(\lambda)}\,p_3(\lambda)\,e^{i\lambda y}\,d\lambda$$

where according to the Meixner condition

$$p(\lambda) = O(\lambda^{3,5}), \quad \lambda \to \infty \qquad (3.7)$$

Furthermore, the boundary conditions (1.3) and (1.4) assume no jump in the functions $q_y(x, 0)$, $q_{yx}(x, 0)$ at $x = 0$ (continuity of the plate displacements and of the angles of turning at the points of attachement to the ribs). We have

$$[q_y(0, 0)] = -2r \int \frac{i\lambda p(\lambda)}{l(\lambda)}\,d\lambda = 0 \qquad (3.8)$$

$$[q_{yx}(0, 0)] = -2r \int \frac{i\lambda p_3(\lambda)}{l(\lambda)}\,d\lambda = 0$$

Here and henceforth the symbol r denotes regularization. This latter is achieved by extraction of the odd part of the integrand /10/. It can be verified that

$$r \int \frac{\lambda^4\,d\lambda}{l(\lambda)} = -\pi, \quad \int \frac{\lambda^2}{l(\lambda)}\,d\lambda = 0$$

We then find $p_3(\lambda) = i\lambda a_0$. from the second equation in (3.8), where a_0 is an arbitrary constant. Furthermore, we have

$$[q_{yxx}(0, 0)] = -2r \int \frac{i\lambda\gamma^2 p(\lambda)}{l(\lambda)}\,d\lambda$$

$$[q_{yxxx}(0, 0)] = -2a_0 r \int \frac{(i\lambda)^2\gamma^2}{l(\lambda)}\,d\lambda = -2\pi a_0$$

The boundary value problem for the diffraction field finally reduces to the following system of integral equations

$$\int \frac{\Pi(\lambda)\,p(\lambda)}{l(\lambda)}\,e^{i\lambda y}\,d\lambda + a_0 i \sin\alpha \int \frac{i\lambda}{l(\lambda)\,S(\lambda)}\,e^{i\lambda y}\,d\lambda + \qquad (3.9)$$

$$y\beta = g(y) \quad (0 < y < h)$$

$$\int \frac{p(\lambda)}{l(\lambda)}\,e^{i\lambda y}\,d\lambda = 0 \quad (y > h)$$

$$Z_1\left(i\sin\alpha\int\frac{i\lambda p(\lambda)}{l(\lambda)S(\lambda)}d\lambda+a_0\int\frac{(i\lambda)^2\Pi(\lambda)}{\gamma^2 l(\lambda)}d\lambda\right)=-2\pi a_0 \tag{3.10}$$

$$-Z_2\beta=-2r\int\frac{i\lambda\gamma^2 p(\lambda)}{l(\lambda)}d\lambda-4\nu\pi\int\frac{p(\lambda)}{l(\lambda)}\rho(\lambda)\,d\lambda$$

$$r\int\frac{i\lambda p(\lambda)}{l(\lambda)}d\lambda=0$$

Here

$$\beta=r\int\frac{i\lambda\Pi(\lambda)\,p(\lambda)}{l(\lambda)}d\lambda+a_0 i\sin\alpha\int\frac{(i\lambda)^2}{l(\lambda)S(\lambda)}d\lambda \tag{3.11}$$

$$\Pi(\lambda)=\frac{\gamma\,\mathrm{sh}\,\gamma d}{S(\lambda)},\quad \rho(\lambda)=\frac{1}{2\pi}\int\limits_0^h se^{i\lambda s}ds$$

The second equation in (3.9) describes the absence of a pressure jump over the ribs.

The system (3.9), (3.10) should be solved in the class of odd functions $p(\lambda)$ with asymptotic (3.7).

4. The system of integral equations formulated above is successfully reduced to an infinite algebraic system by using the method of factorization /4,5/.

A particular solution of the first equation (3.9) can be found by using the representation (3.4), the general solution contains arbitrary functions with prescribed analyticity properties /5/

$$\frac{\Pi(\lambda)\,p(\lambda)}{l(\lambda)}=\frac{i\lambda}{l(\lambda)S(\lambda)}\zeta(\lambda)+\sigma\rho(-\lambda)+F^+(\lambda)+e^{-i\lambda h}F^-(\lambda)\quad(\lambda\in\Lambda) \tag{4.1}$$

$$\zeta(\lambda)=-a_0 i\sin\alpha+\frac{1}{2\pi}G(\lambda,x_0)+\frac{f_1}{2\pi}c(\lambda)+\frac{f_2}{2\pi}\gamma\,\mathrm{sh}\,\gamma d$$

$$\sigma=\frac{f_2}{Z_2}-\beta,\quad F^{\pm}(\lambda)\in W^{\pm}(1/2)$$

The solution of the second integral equation (3.9) has the form /5/

$$p(\lambda)=\psi(\lambda)\quad(\lambda\in\Lambda) \tag{4.2}$$

where $\psi(\lambda)$ is an odd entire function of order h. Using the oddness of the function $p(\lambda)$ and the Liouville theorem, we find from (4.1)

$$\sigma\rho(\lambda)\,l(-\lambda)+l(\lambda)F^+(\lambda)+l(-\lambda)e^{i\lambda h}F^-(-\lambda)=N(\lambda) \tag{4.3}$$

where $N(\lambda)=b_0\,i\lambda+b_2\,(i\lambda)^3$ is an arbitrary odd polynomial of third degree.

Using the function $p(\lambda)$ from (4.1) and (4.2) and taking account of (4.3), we arrive at a boundary value problem of analytic function theory

$$\frac{\Pi(\lambda)\psi(\lambda)}{l(\lambda)}=\frac{i\lambda}{l(\lambda)S(\lambda)}\zeta(\lambda)+\sigma\left(\rho(-\lambda)-\frac{l(-\lambda)}{l(\lambda)}\rho(\lambda)\right)+ \tag{4.4}$$

$$\frac{N(\lambda)}{l(\lambda)}+e^{-i\lambda h}F^-(\lambda)-e^{i\lambda h}\frac{l(-\lambda)}{l(\lambda)}F^-(-\lambda)\quad(\lambda\in\Lambda)$$

The factorization needed later $\Pi(\lambda)=\Pi^+(\lambda)\Pi^-(\lambda)$, where $\Pi^{\pm}(\lambda)\in W^{\pm}(-1/2)$ is constructed in /4/, for instance. Using this partition, we convert the boundary value problem (4.4) to the form

$$\frac{F^-(\lambda)}{\Pi^-(\lambda)}=\frac{\Pi^+(\lambda)\psi(\lambda)}{l(\lambda)}e^{i\lambda h}-\frac{i\lambda\Pi^+(\lambda)}{l(\lambda)\gamma\,\mathrm{sh}\,\gamma d}e^{i\lambda h}\zeta(\lambda)- \tag{4.5}$$

$$\sigma\left(\rho(-\lambda)-\frac{l(-\lambda)}{l(\lambda)}\rho(\lambda)\right)e^{i\lambda h}\frac{\Pi^+(\lambda)}{\Pi(\lambda)}-\frac{N(\lambda)\Pi^+(\lambda)}{l(\lambda)\Pi(\lambda)}e^{i\lambda h}+$$

$$e^{2i\lambda h}\frac{l(-\lambda)}{l(\lambda)}\frac{\Pi^+(\lambda)}{\Pi(\lambda)}F^-(-\lambda)\quad(\lambda\in\Lambda)$$

The left side of the last relationship belongs to the class $W^-(3/2)$, while the right side has simple poles above Λ at the points $\lambda=i\Gamma_n\Gamma_n=((\pi n)^2-k^2)^{1/2}\ (n\geqslant 0)$. By the generalized Liouville theorem

$$\frac{F^-(\lambda)}{\Pi^-(\lambda)}=\sum_{n\geqslant 0}\frac{c_n}{\lambda-i\Gamma_n} \tag{4.6}$$

with arbitrary constants c_n.

There remains to require that the relation (4.5) define the entire function $\psi(\lambda)$. Expressing it from (4.5) and (4.6), we arrive at the necessity to eliminate the possible poles at the points $\lambda=i\Gamma_n$. Equating corresponding residues to zero, we obtain a system of equations for the constants $c_n\ (n\geqslant 0)$

$$c_n + \sum_{p\geqslant 0} A_{np}c_p + r_{n1}a_0 + r_{n2}(b_0 - k^2 b_2) + r_{n3}b_2 = \sigma_n^{\circ} \tag{4.7}$$

$$A_{np} = \exp(-2\Gamma_n h)\frac{L_n^{\circ}}{L_n}\frac{\Pi_n^2 S_n}{\varepsilon_n d\Gamma_n(\Gamma_n+\Gamma_p)}, \quad \varepsilon_n = \begin{cases} 1, & n\geqslant 1 \\ 2, & n=0\end{cases}$$

$$\Pi_n = \Pi^+(i\Gamma_n)$$

$$L_n = -l(i\Gamma_n), \quad L_n^{\circ} = l(-i\Gamma_n), \quad S_n = 1-\cos\alpha(-1)^n$$

$$r_{n1} = -\sin\alpha\frac{\Pi_n(-1)^n}{\varepsilon_n dL_n}\exp(-\Gamma_n h)$$

$$r_{n2} = \frac{\Pi_n S_n}{i\varepsilon_n dL_n}\exp(-\Gamma_n h)$$

$$r_{n3} = r_{n2}\left(\left(\frac{\pi n}{d}\right)^2 + \frac{\pi}{\Gamma_n}(\rho_n L_n^{\circ} + \rho(-i\Gamma_n)L_n)\right), \quad \rho_n = \rho(i\Gamma_n)$$

$$\sigma_n^{\circ} = -\frac{\Pi_n\tau_n(x_0)}{2\pi i\varepsilon_n dL_n}\exp(-\Gamma_n h)$$

$$\tau_n(x_0) = \cos\frac{\pi n x_0}{d}(1-e^{-i\alpha}(-1)^n) - f_1(1-e^{i\alpha}(-1)^n)$$

(we omit the detail associated with evaluating the quantity σ; it turns out that $\sigma = \pi b_2$, $\beta = f_2/Z_2 - \pi b_2$).

In addition to the unknowns c_n, the quantities a_0, b_0, b_2 enter into the system (4.7), and the contact conditions (3.10) must be involved in their determination. The integrals occurring here are evaluated by means of residues, where the identity

$$r\int i\lambda\left(\varphi(-\lambda) - \frac{l(-\lambda)}{l(\lambda)}\varphi(\lambda)\right)d\lambda = -2\nu\int\frac{i\lambda\varphi(\lambda)}{l(\lambda)}d\lambda$$

valid under the condition of the existence of the integral on the right, is used systematically for regularization. Hence, the system of contact equations takes the form (summation over $n\geqslant 0$)

$$s_{m1}a_0 + s_{m2}(b_0 - k^2 b_2) + s_{m3}b_2 + \sum\omega_{mn}c_n = \sigma_m \quad (m=1,2,3) \tag{4.8}$$

$$s_{11}d = -\frac{2\pi d}{Z_1} - \sum\frac{\Gamma_n}{\varepsilon_n L_n}(1+(-1)^n\cos\alpha)$$

$$s_{12}d = 2\pi i\sin\alpha\sum\frac{\Gamma_n}{\varepsilon_n L_n}(-1)^n$$

$$s_{13}d = 2\pi i\sin\alpha\sum\frac{t_n}{\varepsilon_n L_n}, \quad t_n = \left(\frac{\pi n}{d}\right)^2\Gamma_n + 2\pi\nu\rho_n$$

$$\sigma_1 d = -i\sin\alpha\sum\frac{\Gamma_n(-1)^n}{\varepsilon_n L_n S_n}\tau_n(x_0) - \frac{f_1 d}{Z_1}$$

$$s_{21} = s_{12}, \quad s_{22}d = -2\pi\sum\frac{\Gamma_n S_n}{\varepsilon_n L_n}$$

$$s_{23}d = -2\pi\sum\frac{t_n S_n}{\varepsilon_n L_n}, \quad \sigma_2 d = \sum\frac{\Gamma_n}{\varepsilon_n L_n}\tau_n(x_0)$$

$$s_{31} = s_{13}, \quad s_{32} = s_{23}, \quad s_{33}d = -\pi dZ_2/2 -$$

$$8\pi^2\nu\sum\frac{\rho_n S_n}{L_n}\left(\frac{\pi n}{d}\right)^2 + \pi(1-\cos\alpha) - 2\pi\sum\frac{\Gamma_n k_0^4 + \nu}{L_n}S_n -$$

$$\pi d\nu\left(\frac{2}{d}\sum\frac{S_n}{\varepsilon_n\Gamma_n^5}\left(1-\exp(-\Gamma_n h) - \Gamma_n h\exp(-\Gamma_n h) -\right.\right.$$

$$\left.\left.\frac{(\Gamma_n h)^2}{2}\right) + \frac{\cos\alpha - \cos kd}{k\sin kd}\frac{h^3}{3}\right) - 4\pi^3\nu\sum\frac{\rho_n^2 S_n L_n^{\circ}}{\varepsilon_n\Gamma_n L_n}$$

$$\sigma_3 d = -\sum\frac{t_n}{\varepsilon_n L_n}\tau_n(x_0), \quad \omega_{mp} = 4\pi\nu\sum\frac{r_{nm}}{\Gamma_n+\Gamma_p}$$

$$(m=1,2,3; \ p\geqslant 0)$$

Acceleration of the convergence of the series and products mentioned is achieved by extraction of the principal, slowly convergent part /11/.

5. To prove the equivalence of the infinite system (4.7), (4.8) and the initial boundary value problem, the function $F^-(\lambda)$ and $\psi(\lambda)$ should be introduced according to (4.5) and (4.6) and confirmed as belonging to the required classes of analytic functions, then $p(\lambda)$ is introduced according to (4.1) and satisfaction of the integral equations (3.9) and (3.10) is

confirmed. The equivalence of the latter to the initial boundary value problem is verified without difficulty.

The system of equations (4.7), (4.8) generates an equation of the second kind in the space of sequences l_2

$$\mathbf{c} + A(k)\,\mathbf{c} = \mathbf{f},\ \mathbf{c} = (a_0, b_0, b_2, c_0, c_1 \ldots) \tag{5.1}$$

where the Hilbert–Schmidt operator $A(k)$ is an analytic operational function of the complex parameter k. From the uniqueness of the solution of the initial boundary value problem for $\operatorname{Im} k > 0$ the discreteness and realness of the set of eigenfrequencies of the problem follows on the basis of the analytic Fredholm theorem /12/.

A solution exists at non-resonance frequencies, is unique, and can be found by the method of reduction /11/. We note also that the substitution $c_n = -r_{n2}b_2 + c_n' \exp(-\Gamma_n h)$ yields a system with exponential convergence for the new unknowns c_n', where $c_n = O(n^{-4/3})$ $(n \to \infty)$.

6. We turn to a study of the structure of the total field $p(x, y)$. Within the comb $(0 < x < d,\ 0 < y < h)$ it is representable in the form

$$p(x, y) = d_0 y (\cos k(x - d) - e^{i\alpha} \cos kx) + \sum \cos \frac{\pi n x}{d} [g_n \exp(-\Gamma_n (h - y)) + h_n \exp(-\Gamma_n y)] \tag{6.1}$$

Without presenting the expressions for the coefficients d_0, g_n, h_n, we note just the asymptotic $g_n = O(n^{-4/3})$ which can be utilized to verify satisfaction of the Meixner condition by using the Euler–Maclauren summation formula.

The field over the ribs $(y > h)$ is a set of planar and quasiplanar waves

$$p(x, y) = \sum_{-\infty}^{\infty} A_n \exp(i(\lambda_n x + \sqrt{k^2 - \lambda_n^2}\, y)) \tag{6.2}$$

The amplitudes A_n are expressed in terms of the solution of the infinite system (4.7), (4.8), particularly for the fundamental waves $p_1 = A_1 \exp(i(\alpha x/d + by))$ we have

$$A_0 = -\nu F \frac{2\pi\alpha}{bd^2\Pi^+(b)} e^{-ibh} \sum_{n \geqslant 0} \frac{c_n}{b - i\Gamma_n}, \quad b = \sqrt{k^2 - \left(\frac{\alpha}{d}\right)^2} \tag{6.3}$$

The total number of waves being propagated over the ribs can be defined as the number of integers in the interval $(-(kd + \alpha)/(2\pi),\ (kd - \alpha)/(2\pi))$. For $2\pi - kd > \alpha > kd$, there are generally no waves being propagated (the sources are applied to the plate "much too often" and compensate each others' radiation to a certain degree).

In the case $\alpha < kd < \pi$ only the fundamental wave p_1 is propagated. If the construction is excited cophasally here $(\alpha = 0)$, then as an analysis of the infinite system shows, the amplitude A_0 agrees with the value calculated by the zero-th approximation, and therefore, taking account of the solidity of the rib does not change the power radiated.

We introduce the means in the period of the energy flux π transported in the vertical direction by the waves p_1, p_{10} (the fundamental wave in the zero-th approximation). We have successively

$$\pi_1 = \frac{1}{2\rho_0\omega} \operatorname{Im}\left(\bar{p}_1 \frac{\partial p_1}{\partial y}\right) = \frac{b}{2\rho_0\omega} |A_0|^2$$

$$\pi_{10} = \frac{b}{2\rho_0\omega} |A_{10}|^2, \quad \pi_{00} = \frac{b}{2\rho_0\omega} |A_{00}|^2$$

$$A_{00} = -\frac{\nu F}{dL_0} \exp\left(-\frac{i\alpha x_0}{d}\right), \quad A_{10} = A_{00}\left(1 + \exp\frac{i\alpha x_0}{d}\left(f_1 + \frac{i\alpha}{d} f_2\right)\right)$$

Here A_{00}, A_{10} are amplitudes of the waves p_{00} and p_{10}, respectively (see (3.1)). We also introduce the normalized flux (in Hz)

$$\tau_1 = 10 \lg(\pi_1/\pi_{00}),\ \tau_0 = 10 \lg(\pi_{10}/\pi_{00})$$

The dependences of τ_1 and τ_0 on the frequency f (in Hz) are displayed in Fig.1 for $x_0 = 0$ (a) and $x_0 = 0.25\,d$ (b), and on the point of application of the force x_0 in Fig.2 $f = 400$ Hz; all the dependence on x_0/d are symmetric relative to the axis $x_0/d = 0.5$). The lines *1* and *2* correspond to the phase $\alpha = 0.1\,kd$, *3* and *4* to $\alpha = 0.5\,kd$, and *5* and *6* to $\alpha = 0.9\,kd$. The lines *1, 3, 5* here refer to the solid-free approximation (τ_0), and *2, 4*, and *6* to the total field (τ_1). Computations were performed for a steel plate of 4cm thickness, reinforced by steel ribs of thickness 3 cm, in water, $d = 60$ cm, $h = 30$ cm.

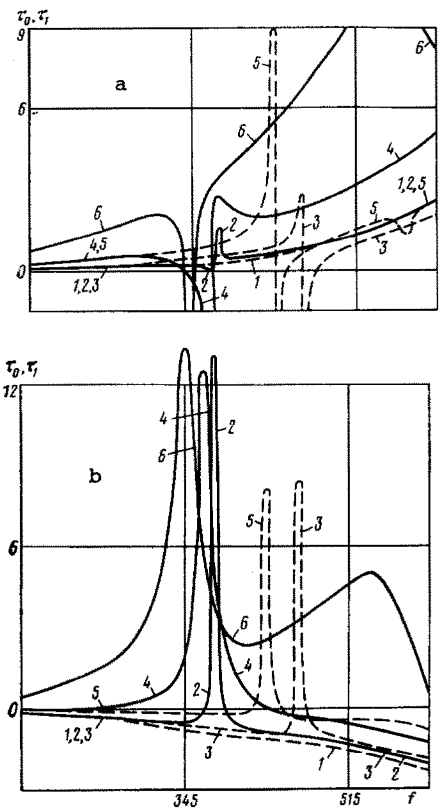

Fig.1

We note that the magnitude of the radiated energy is controlled by the interaction of two factors. On the one hand, attachement of the ribs to the plate and the fluctuating fluid mass result in an increase in system resistance, which specifies a diminution in the energy introduced. But on the other hand, re-reflection of the waves by each rib is an additional radiation source since a redistribution of the total energy here occurs between the structure vibration energy and the energy emitted into the fluid.

The radiated field undergoes sharp changes depending on the frequency and points of application of the force, changes in x_0 can here result in replacement of the radiation maximums by the minimums, and conversely. A similar dependence of the radiated power on x_0 was also noted in the case of a single rib /2/. The dependences of damped sinusoid type occurring there are here, roughly speaking, combined for each "rib-force" pair to generate quite complex curves.

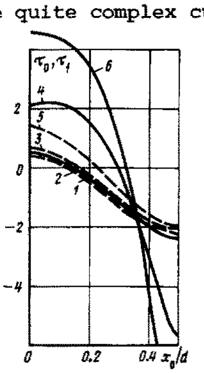

Fig.2

We turn to a study of the characteristic frequencies at which sharp changes occur in the radiated field. The frequencies mentioned are independent of x_0, they are determined solely by the Fredholm denominator of equation (5.1), in other words, the determinant of the infinite system of equations diminishes abruptly at these frequencies. Of interest is the dynamics of the frequencies on the parameters of the problem. As is known, an increase in the wave number $\varkappa$ of the plate in the fluid as compared to a vacuum is due to the influence of the apparent mass of the fluid. The characteristic frequencies in the problem of the zero-th approximation are determined from the equations $|2\pi n + \alpha| = \varkappa d$ $(n = 0, \pm 1, \ldots)$. Only one solution $f = 565$ Hz for $n = -1, \alpha = 0.9\,kd$. is in the rated band. A bump is noted around this frequency in all curves 5.

However, the mentioned frequency is the upper limit of the possible characteristic frequencies since the addition of ribs to the plate also produces an apparent mass that increases the effective wave number of the structure $\varkappa^\circ$. Consequently, the characteristic frequencies are roots of the equation $2\pi - \alpha = \varkappa^\circ d$ and shift towards diminution, where the shift grows with the phase α.

Taking account of the solidity of the ribs results in an increase in the apparent mass because of the fluid vibrating together with the ribs. This explains the leftward shift in frequency of the curves *2*, *4* and *6* relative to curves *1*, *3* and *5*. The conception of an apparent mass also explains the inverse dependence of the characteristic frequencies on the ratio θ of the rib mass to the mass of the plate span observed in the computations.

The effect of the solidity also depends on the rib height, for $h/d \leqslant 0.25$ the solidity cannot be taken into account. Furthermore, this effect depends on the degree of phasing of the set, for $\alpha \leqslant 0.1kd$ it too cannot be taken into account. Qualitatively this circumstance is explained by involving the direction of the wave vector $\mathbf{n}$ of the wave being propagated: $\mathbf{n} = (t, (1-t^2)^{1/2})$, $t = 0.1, 0.5, 0.9$. As the phase factor t grows, the propagation direction becomes more and more shallow so that the ribs surfaces apparently are involved to a great extent in field formation, while the role of the solidity is less for small phases.

Finally, we discuss the influence of the rib mass on the magnitude of the power emitted. Taking account of the solidity does not yield a correction greater than 2 dB for values of the parameter $\theta \geqslant 10$ (very heavy ribs). If the force acts the rib $(x_0 = 0)$, the radiation gain band with respect to frequency is quite small since the heavy ribs swing weakly, retarding the radiation of the closely located forces. For $\theta \geqslant 6$ the frequency dependences already cease to be resonant in nature. Conversely, for small θ the light ribs can swing strongly involving the attached fluid in the vibrations, hence the role of the solidity increases.

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